N11/5/MATHL/HP3/ENG/TZ0/SP

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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Friday 4 November 2011 (morning)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 15]*

The weight of tea in *Supermug* tea bags has a normal distribution with mean 4.2 g and standard deviation 0.15 g. The weight of tea in *Megamug* tea bags has a normal distribution with mean 5.6 g and standard deviation 0.17 g.

(a) Find the probability that a randomly chosen *Supermug* tea bag contains more than 3.9 g of tea. *[2 marks]*

(b) Find the probability that, of two randomly chosen *Megamug* tea bags, one contains more than 5.4 g of tea and one contains less than 5.4 g of tea. *[4 marks]*

(c) Find the probability that five randomly chosen *Supermug* tea bags contain a total of less than 20.5 g of tea. *[4 marks]*

(d) Find the probability that the total weight of tea in seven randomly chosen *Supermug* tea bags is more than the total weight in five randomly chosen *Megamug* tea bags. *[5 marks]*

2. *[Maximum mark: 7]*

Neil wants the opinion of teachers on a proposal to change the Mathematics HL curriculum. A questionnaire is sent to a large number of teachers asking for their opinions on the proposal. Of the 200 replies he receives, 160 are in favour of the proposal. Assume that these teachers are a random sample from the population.

(a) Test, at the 5 % level, the hypothesis that the proportion of the population in favour of the proposal is 0.75 against the alternative that it is more than 0.75. *[4 marks]*

(b) Find a 95 % confidence interval for the proportion of the population in favour of the proposal. *[3 marks]*

3. *[Maximum mark: 11]*

The random variable X represents the lifetime in hours of a battery. The lifetime may be assumed to be a continuous random variable X with a probability density function given by $f(x) = \lambda e^{-\lambda x}$, where $x \geq 0$.

(a) Name this distribution and state its mean. *[1 mark]*

(b) Find the cumulative distribution function, $F(x)$, of X. *[3 marks]*

(c) Find the probability that the lifetime of a particular battery is more than twice the mean. *[2 marks]*

(d) Find the median of X in terms of λ. *[3 marks]*

(e) Find the probability that the lifetime of a particular battery lies between the median and the mean. *[2 marks]*

4. *[Maximum mark: 16]*

The random variable X is believed to be modelled by $\mathrm{B}(5, 0.5)$. A random sample of size 100 is taken and the observed frequencies are given in the table below.

Value of X	0	1	2	3	4	5
Observed frequency	2	15	s	$69 - s$	12	2

A χ^2 goodness of fit test is carried out on these data.

(a) State the null and alternative hypotheses. *[1 mark]*

(b) Evaluate the χ^2 statistic in the form $as^2 + bs + c$. *[8 marks]*

(c) Find the range of values of s that would result in the null hypothesis being accepted at the 10 % level. *[7 marks]*

5. *[Maximum mark: 11]*

The continuous random variable U has a uniform distribution on $[0, 1]$. The random variable X is defined as follows:

$$X = 2U \text{ when } U \leq \frac{3}{4}$$

$$X = 4U \text{ when } U > \frac{3}{4}.$$

(a) (i) Explain why X cannot take values in the interval $\frac{3}{2} < X \leq 3$.

(ii) Find $\mathrm{P}\left(0 \leq X \leq \frac{3}{2}\right)$.

(iii) Find $\mathrm{P}(3 < X \leq 4)$. *[6 marks]*

(b) Find the lower quartile of X. *[3 marks]*

(c) Find $\mathrm{E}(X)$. *[2 marks]*